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ANALYSIS OF NUMERICAL SOLUTIONS OF A HEREDITARY DEFORMABLE SYSTEM

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ABSTRACT

An investigation of a viscoelastic material damping effect is studied on an example of plenum air-cushion craft model. A numerical investigation was conducted to determine the vertical response characteristic of an open plenum air-cushion structure. The pure vertical motion of an air-cushion structure is investigated using a non-linear mathematical model; this incorporates a simple model to account the hereditary deformable characteristic of the material.

KEYWORDS: Viscoelasticity, Hereditary Deformable, Air-Cushion & Integro-Differential Equation

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Original Article

1. INTRODUCTION

The term “Viscoelastic” material has quite a broad meaning. For example, in the literature [1 to 6] there is a use of this term if an equation of motion includes a viscous damping term; i. e., the equation of motion is written in terms of current instant values of displacement, velocity, and acceleration. In this study, the equation of motion will include the integral term and history of strain is required for its formulation. Materials yielding such a constitutive relation (requiring history) are also called viscoelastic, but the term “hereditary” materials will be used in this study to distinguish them.

Hereditary properties are present in any composite material [1, 2 and 3]. The definition of the hereditary medium will give below. At this stage, we just note that the class of viscoelastic materials includes a subclass the hereditary materials; i. e., these two terms are not identical.

In this study, numerical integration is applied for solution of the integro-differential equation. According to the correspondence (Volterra) principle, a solution of the viscoelastic problem can be obtained from a solution of the corresponding elastic problem be the replacement of elasticity constants by their hereditary analogs (integral operators).

In this study, the use of exponential terms for relaxation kernels is utilized.

2. MODEL OF HEREDITARY DYNAMIC SYSTEM

Consider the disturbed motion of system near equilibrium with a view to sustaining only vertical motion.

As to the constitutive relations, there are different models in use. Different models of viscoelastic material are discussed in references [4, 5, and 6].

In this study, we build a model, which based on a constitutive law of the form:

$$a\dot{\sigma}+\sigma=b\dot{y}+cy\,, \tag{1}$$

where

$$\sigma=F\Delta P,\quad \theta=\frac{\rho_0 V_0}{G_0},\quad a=\frac{2\theta P_{U_0}}{3nP_0},\, b=\frac{2\theta P_0}{3H}=\frac{2mg\theta}{3H},\quad c=\frac{2P_{U_0}F}{3h_0}=\frac{2mg}{3h_0}$$

In the references [1-6] are listed the different standard models of viscoelasticity. Therefore, the viscoelastic system (1) have the properties of complex viscoelastic suspension and the relation between σ and *y*, can be written as a hereditary type of exponential kernel of relaxation

$$\sigma=\frac{b}{a}\left[y(t)-\int_0^t R(t-\tau)y(\tau)d\tau\right] \tag{2}$$

where

$$R(t-\tau)=\left(\frac{1}{a}-\frac{c}{b}\right)e^{-\frac{1}{a}(t-\tau)} \tag{3}$$

However the model of the viscoelastic system (2) with an exponential kernel (3) includes creep strain, and stress relaxation, but have one major weakness such as $\dot{y}(t)$ in initial time has the final values, and do not fulfilled with experiment. This disadvantage easily overcome by use of weak-singularity features of the relaxation kernel (3) following from [3]:

$$R(t-\tau)=\left(\frac{1}{a}-\frac{c}{b}\right)e^{-\frac{1}{a}(t-\tau)}(t-\tau)^{a-1},\quad 0<a<1 \tag{4}$$

The fact, when the model of the system is made of composite materials [4], then the relation between σ and *y* must obey the law of hereditary non-linear theory viscoelasticity such:

$$\sigma=\frac{b}{a}\left[\left(y(t)-\gamma y^3(t)\right)-\int_0^t R(t-\tau)\left[y(\tau)-\gamma y^3(t)\right]d\tau\right] \tag{5}$$

The relation (5) is fairly common because in particularity can be obtained by the standard model of a viscoelastic body with kernels of relaxation (3). If considered that nonlinearity is $\gamma=0$, will be obtained the known linear relation of the hereditary theory.

The equation of motion will be:

$$m\ddot{y}(t)+\sigma=0 \tag{6}$$

Substituting (5) into (6) is built weak singular integro-differential equation of nonlinear hereditary deformable system. This equation in dimensionless coordinates can be written in text form

$$\ddot{U}(t)+\left[U(t)-\gamma U^{3}(t)\right]-(\mu-N)\int_{0}^{t}\Gamma(t-\tau)\left[U(\tau)-\gamma U^{3}(\tau)\right]d\tau=0 \tag{7}$$

where $\mu=\frac{3nP_0}{2P_{U_0}}$, $N=\frac{H}{h_0}$, $\Gamma(t-\tau)=A_0e^{-\beta(t-\tau)}(t-\tau)^{a-1}$. The initial conditions are

$$U(0)=U_0,\quad \dot{U}(0)=\dot{U}_0 \tag{8}$$

The equation (7) with initial conditions (8) represents a mathematical model of the hereditary deformable system.

3. NUMERICAL EXAMPLE

The results of calculations of steady-state responses according to the expression of previous sections are presented below.

As the example will be calculated the vertical dynamic response of plenum air cushion craft. Both theoretical and experimental research [9, 10, 12-14] has been performed to study the vertical motion and /or stability of various air cushion design configurations. The theoretical works [10] have been based on linear approaches to analyses air cushion vertical responses. Since air cushions are nonlinear, the application of linear analysis may be insufficient to predict fully the air cushion dynamic response behavior. Furthermore, any nonlinearity in air cushion response will have a direct bearing.

In this section presents the results of a numerical investigation to determine the dynamic behavior (in vertical) of a simple plenum air cushion suspension system in response to steady-state disturbances. The linear and nonlinear viscoelastic behavior of the system is examined.

An air-cushion craft structure is loaded by a vertical pressure load P, which is graphically presented in Figure 1 for a mass flow rate Q (Q_{in} and Q_{out}) to cushion.

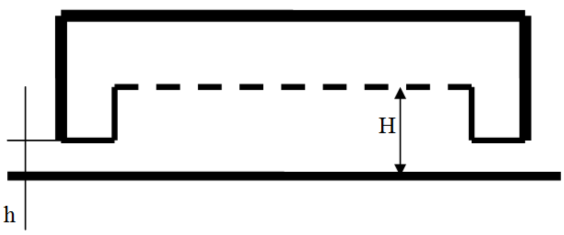

Figure 1: A Plenum Air Cushion scheme

From this Figure 1 geometry of the air cushion represented by h-the distance between the ground and a lower surface of the air cushion body and H is the average height of the air cushion. Motion parameters of the air cushion can be described by

$$V=V_0(1-\frac{y}{H})\text{ and }h=h_0\left(1-\frac{y}{h_0}\right) \tag{9}$$

where $y=h_0-h$ (displacement of the air cushion).

Relation between inner volume pressure and density of air equal to $p\rho^{n} = const$, where n is the polytrophic parameter of air.

By using equation (7) with initial conditions (8) is built a mathematical model of the air cushion motion and stability problem. There are two unknowns: $U(t)$ and N. Therefore, it is needed to find the values of *N* on which system is self-vibrating.

Exact solution of the equation (7) is not possible, so the numerical simulation of this equation with initial conditions (8) will be done by the method, offered in [7]

$$U_n = U_0 + t_n U_0 - \sum_{j=1}^{n-1} a_j (t_n - t_j)\left[U_j - \gamma U_j^3 - \frac{(\mu - N)A_0}{a}\sum_{k=0}^{j} B_k e^{-\beta t_n}\left[U_{j-k} - \gamma U_{j-k}^3\right]\right] \quad (10)$$

where

$$t_n = n\Delta t, \quad a_j = \Delta t, \quad a_0 = a_n = \Delta t/2$$
$$j = 1, n-1; \quad k = 1, j-1$$
$$B_0 = \Delta t^a/2; \quad B_j = \Delta t^a\left[j^a - (j-1)^a\right]/2$$
$$B_k = \Delta t^a\left[(k+1)^a - (k-1)^a\right]/2$$

To solve equation (10) is setup technical and kinematical characteristic of the air cushion as followed: h_0 – distance between the lower surface of the air cushion and ground equal [0.02-0.12] meter; average height of the air cushion height is 1.25 meter; $n = 1.4$ and pressure ratio is 1.2. Rheological and nonlinearity parameters are varied for calculation of elastic, viscoelastic, and linear visa versa nonlinear cases.

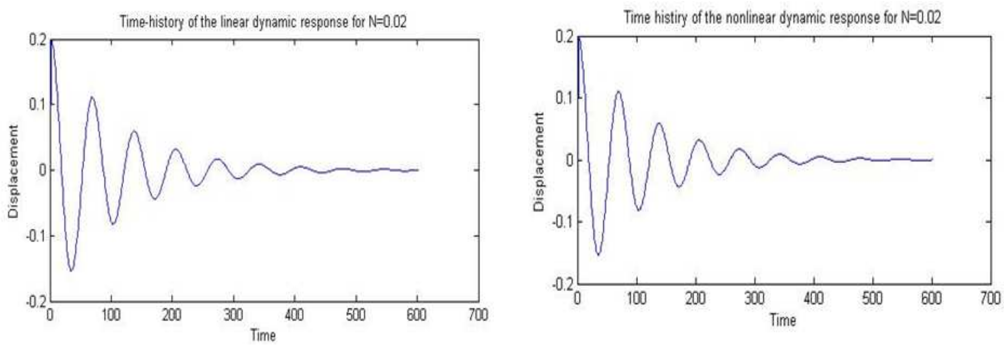

Figure 2: Time History of the Vertical Dynamics Responses for N=0.02
(a) Linear Case and (b) Nonlinear Case

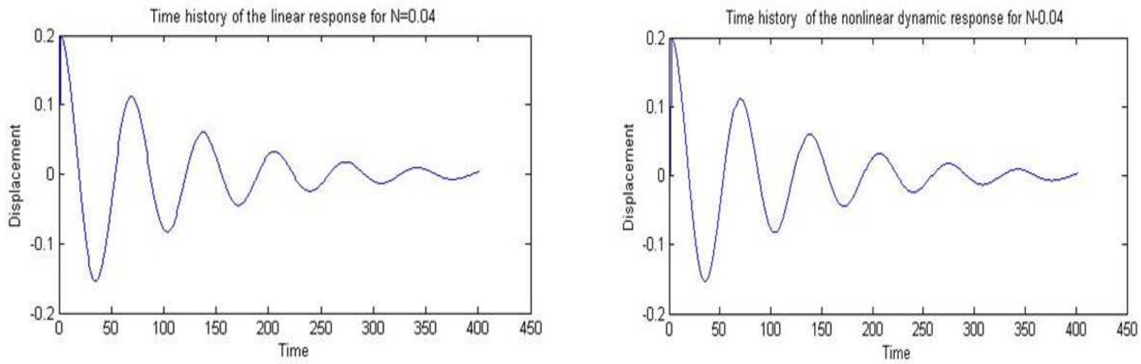

Figure 3: Time History of the Vertical Dynamics Responses for N=0.04
(a) Linear Case and (b) Nonlinear Case

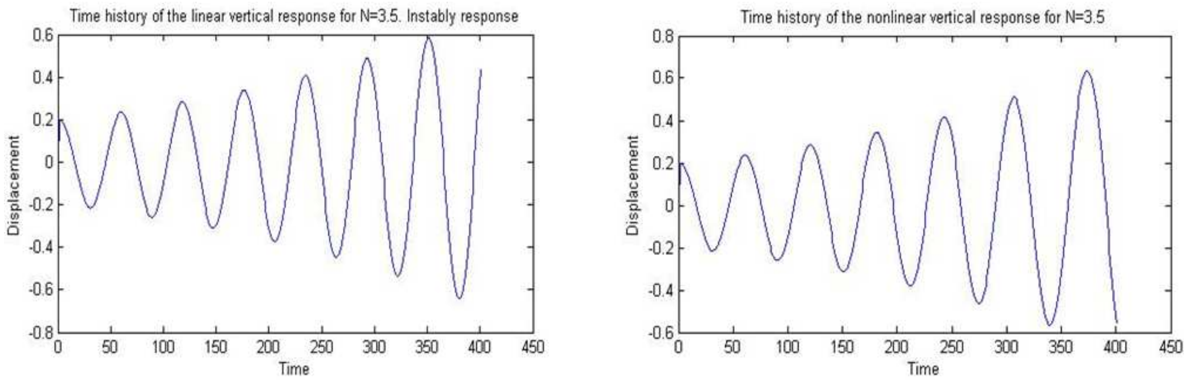

Figure 4: Time History of the Vertical Dynamics Responses for N=3.5
(a) Linear Case and (b) Nonlinear Case

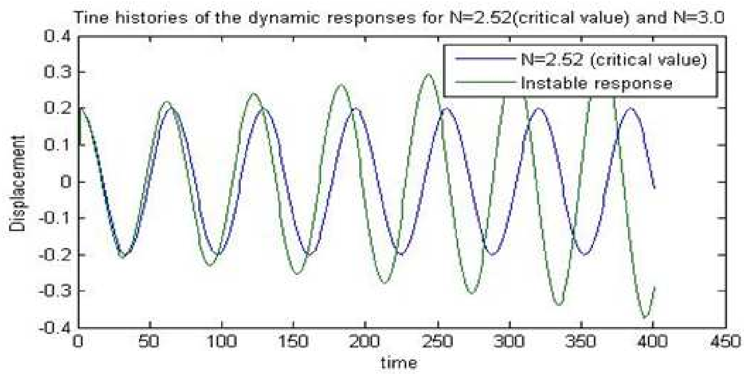

Figure 5: Comparison of Dynamic Responses

From the analysis of solutions of the equation (7) follows that $\mu - N > 0$ in both linear and non-linear case is taking place damped oscillatory process. The damping speed and dissipation characteristics of the system are significantly dependent on the rheological parameters A_0, β and a. When the system has smaller the singularity parameter a of the structure material, then damping properties of this material is higher. The self-vibrating is occurs only if $\mu - N < 0$. Effect of physical non-linearity's and rheological parameters on the critical speed N_{cr} is not difficult to show, through computer experiment based upon an algorithm (9).

4. CONCLUSIONS

Application of the numerical integration method to the hereditary deformable problem is demonstrated. In this study, the numerical solution in the time domain for the dynamic problem (stability and flutter problem) have been presented.

The constitutive relation (stress-strain) was used in form of a hereditary law with the relation kernel represented by Abelian type function.

Numerical experiments for a problem of system bending vibration under force have been conducted. Finally, an understanding of viscoelastic phenomena may be exploited to damping in a manner that improves stability the of the craft.

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